1/22/20

Accumulation Functions (Summary)

1) i - simple $\Rightarrow a(t) = (1 + i \cdot t)$ t - in years

2) i - peir $\Rightarrow a(t) = (1+i)^t$ t - # of periods

3) d - simple $\Rightarrow a(t) = (1 - dt)^{-1}$ t - in years

4) d - pedr $\Rightarrow a(t) = (1-d)^{-t}$ t - # of periods

5) δ_t - foi $\Rightarrow a(t) = e^{\int_0^t \delta_r \, dr}$

(a) $\delta_t = \delta \Rightarrow a(t) = e^{\delta t}$

(b) $\delta_t = c \cdot \dfrac{f'(t)}{f(t)} \Rightarrow a(t) = \left[\dfrac{f(t)}{f(0)}\right]^c$

Examples (See next pages)

An account credits interest using a simple discount rate, d. A deposit of X at time $t = 0$ accumulates to $2X$ at time $t = 5$. $\underbrace{\text{Determine the time}}_{=n}$ at which the account will have $4X$.

(A) 7.5

(B) 8.0

(C) 8.5

(D) 9.0

(E) 10.0

$$a(t) = (1 - dt)^{-1}$$

$$X \cdot a(5) = 2X \Rightarrow a(5) = 2 = (1 - 5d)^{-1}$$

$$\Rightarrow d = 0.1$$

$$X \cdot a(n) = 4X \Rightarrow a(n) = 4 = (1 - .1n)^{-1}$$

$$\Rightarrow n = 7.5 \quad \text{A}$$

An account credits interest using a simple interest rate, i, for the first three months, then a discount rate of 6%, convertible monthly, for the next nine months. Thereafter, the account credits interest using an interest rate of i, payable quarterly, which is equivalent to an annual effective discount rate, d. An initial deposit of ~~7395~~ 8071 accumulates to 10,000 at the end of five years. Determine d.

$i = i^{(4)}$ $\qquad .06 = d^{(12)}$

(A) 3.8%

(B) 3.9%

(C) 4.0%

(D) 4.1%

(E) 4.2%

$$10000 = 8071 \cdot \left(1 + i \cdot \tfrac{1}{4}\right) \cdot \Big(\underbrace{1 - \tfrac{.06}{12}}_{=.995}\Big)^{-9} \cdot \left(1 + \tfrac{i}{4}\right)^{16}$$

$$\Longrightarrow i = \left[\left(\frac{10000\,(.995)^9}{8071}\right)^{1/17} - 1\right] \cdot 4 = 0.04\ldots$$

$$i^{(4)} = .04 \Longrightarrow \text{aedr} = d = ?$$

$$\text{aaf} = \left(1 + \tfrac{.04}{4}\right)^4 = (1-d)^{-1} \Longrightarrow d = .039\ldots \quad \text{B}$$